

# **SOALAN RAMALAN**

## *MATEMATIK TAMBAHAN*

### *KERTAS 2*

**FUNGSI TRIGONOMETRI**  
*TRIGONOMETRY FUNCTION*



FUNGSI TRIGONOMETRI  
YIK



(a) Buktikan  $\sin^2 \theta + \cos^2 \theta = 1$ .

*Prove  $\sin^2 \theta + \cos^2 \theta = 1$ .*

(b) Lakarkan graf  $y = 2|\sin 2x| + 1$  untuk  $0 \leq x \leq 2\pi$ .

*Sketch the graph of  $y = 2|\sin 2x| + 1$  for  $0 \leq x \leq 2\pi$ .*



(a) (i) Buktikan bahawa  $\frac{4}{2 \cot \theta - 2 \tan \theta} = \tan 2\theta$  .

*Prove that  $\frac{4}{2 \cot \theta - 2 \tan \theta} = \tan 2\theta$  .*

[ 2 markah]  
[2 marks]

(ii) Seterusnya, selesaikan persamaan  $\frac{4}{2 \cot \theta - 2 \tan \theta} = \frac{\sqrt{3}}{2}$  untuk  $0^\circ \leq \theta \leq 270^\circ$  .

*Hence, solve the equation  $\frac{4}{2 \cot \theta - 2 \tan \theta} = \frac{\sqrt{3}}{2}$  for  $0^\circ \leq \theta \leq 270^\circ$  .*

[3 markah]  
[3 marks]

(b) Lakarkan graf  $4y + |8 \sin 2x| - 8 = 0$  bagi  $0 \leq x \leq \frac{3}{2}\pi$  . Seterusnya, cari julat nilai  $k$  bagi persamaan  $8 - |8 \sin 2x| - 4k = 0$  supaya persamaan itu hanya mempunyai 6 penyelesaian untuk  $0 \leq x \leq \frac{3}{2}\pi$  .

*Sketch the graph of  $4y + |8 \sin 2x| - 8 = 0$  for  $0 \leq x \leq \frac{3}{2}\pi$  . Hence, find the range of values of  $k$  for the equation  $8 - |8 \sin 2x| - 4k = 0$  so that the equation has only 6 solutions for  $0 \leq x \leq \frac{3}{2}\pi$  .*

[5 markah]  
[5 marks]

FUNGSI TRIGONOMETRI  
MRSM



(a) Buktikan  $\frac{\cot^2 x}{1 + \cot^2 x} = \cos^2 x$ . [2 markah]

*Prove that*  $\frac{\cot^2 x}{1 + \cot^2 x} = \cos^2 x$ . [2 marks]

(b) (i) Lakarkan graf  $y = \frac{2}{\cos x} \left( \frac{\cot^2 x}{1 + \cot^2 x} \right) + 1$  untuk  $0 \leq x \leq 2\pi$ . [3 markah]

*Sketch the graph of*  $y = \frac{2}{\cos x} \left( \frac{\cot^2 x}{1 + \cot^2 x} \right) + 1$  for  $0 \leq x \leq 2\pi$ . [3 marks]

(ii) Seterusnya, dengan menggunakan paksi yang sama, lakarkan satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan

$$\frac{2}{\cos x} \left( \frac{\cot^2 x}{1 + \cot^2 x} \right) = \frac{x}{\pi} - 1 \text{ untuk } 0 \leq x \leq 2\pi.$$

Nyatakan bilangan penyelesaian itu. [3 markah]

*Hence, using the same axes, sketch a suitable straight line to find the*

*number of solutions for the equation*  $\frac{2}{\cos x} \left( \frac{\cot^2 x}{1 + \cot^2 x} \right) = \frac{x}{\pi} - 1$

*for*  $0 \leq x \leq 2\pi$ .

*State the number of solutions.* [3 marks]

FUNGSI TRIGONOMETRI  
SABK S1



(a) Buktikan  $\sin\left(4\theta - \frac{\pi}{3}\right) + \sin\left(4\theta + \frac{\pi}{3}\right) = \sin 4\theta$ .

[2 mark]

*Prove that*  $\sin\left(4\theta - \frac{\pi}{3}\right) + \sin\left(4\theta + \frac{\pi}{3}\right) = \sin 4\theta$ .

[2 mark]

(b) Seterusnya,

*Hence,*

(i) lakar graf  $y = \sin\left(4\theta - \frac{\pi}{3}\right) + \sin\left(4\theta + \frac{\pi}{3}\right)$  untuk  $0^\circ \leq \theta \leq \pi$ ,

*sketch the graph of*  $y = \sin\left(4\theta - \frac{\pi}{3}\right) + \sin\left(4\theta + \frac{\pi}{3}\right)$  *for*  $0^\circ \leq \theta \leq \pi$ ,

[3 mark]

[3 mark]

(ii) cari nilai  $m$  jika persamaan  $2\pi \sin 4\theta \cos \frac{\pi}{3} = m\pi - \pi$  bersilang pada titik  $\left(\frac{\pi}{2}, 0\right)$ .

*find the value of*  $m$  *if the equation*  $2\pi \sin 4\theta \cos \frac{\pi}{3} = m\pi - \pi$  *intersects at point*  $\left(\frac{\pi}{2}, 0\right)$

**FUNGSI TRIGONOMETRI**  
**SABK S3**



(a) Lakar graf bagi  $y = -3 \sin \frac{3}{2}x$  untuk  $0 \leq x \leq 2\pi$ . [4 markah]

*Sketch the graph of  $y = -3 \sin \frac{3}{2}x$  for  $0 \leq x \leq 2\pi$ .* [4 marks]

(b) Seterusnya, dengan menggunakan paksi yang sama, lakar satu graf yang sesuai untuk mencari bilangan penyelesaian bagi persamaan  $\frac{\pi}{x} + 3 \sin \frac{3}{2}x = 0$  untuk  $0 \leq x \leq 2\pi$ .

Nyatakan bilangan penyelesaian itu. [3 markah]

*Hence, using the same axes, sketch a suitable graph to find the number of solutions for the*

*equation  $\frac{\pi}{x} + 3 \sin \frac{3}{2}x = 0$  for  $0 \leq x \leq 2\pi$ .*

*State the number of solutions.* [3 marks]

**FUNGSI TRIGONOMETRI**  
**SABK S2**



12) Jika  $x = \cos 45^\circ$  dan  $y = \sin 25^\circ$ , nyatakan dalam sebutan  $x$  dan/atau  $y$   
If  $x = \cos 45^\circ$  and  $y = \sin 25^\circ$ , state in terms of  $x$  and/or  $y$

(i)  $\cos 50^\circ$

$\cos 50^\circ$

(ii)  $\sin 20^\circ$

[4 markah]

[4 marks]

(b) Selesaikan persamaan  $2 \sec x = 2 \tan x - \cot x$  bagi  $0^\circ \leq x \leq 360^\circ$ .

Solve the equation  $2 \sec x = 2 \tan x - \cot x$  for  $0^\circ \leq x \leq 360^\circ$ .

[4 markah]

[4 marks]

**FUNGSI TRIGONOMETRI**  
**SABK TRIAL**



(a) Lakarkan graf  $y = \left| -\frac{3}{2} \cos 2x \right| - 1$  untuk  $0 \leq x \leq \frac{3}{2}\pi$ . [4 markah]

*Sketch the graph of  $y = \left| -\frac{3}{2} \cos 2x \right| - 1$  for  $0 \leq x \leq \frac{3}{2}\pi$ . [4 marks]*

(b) Seterusnya, dengan menggunakan paksi yang sama, lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan  $3\pi \left| -\frac{3}{2} \cos 2x \right| - 4x = -2x$  untuk

$0 \leq x \leq \frac{3}{2}\pi$ . Nyatakan bilangan penyelesaian itu. [4 markah]

*Hence, using the same axes, sketch a suitable straight line to find the number of solutions*

*for the equation  $3\pi \left| -\frac{3}{2} \cos 2x \right| - 4x = -2x$  for  $0 \leq x \leq \frac{3}{2}\pi$ . State the number of solutions. [4 marks]*

**FUNGSI TRIGONOMETRI**  
**MIMS S1**



- (a) Lakar graf bagi  $y = |4 \sin 2x|$  untuk  $0 \leq x \leq \frac{3}{2}\pi$ . [3 markah]

*Sketch the graph of  $y = |4 \sin 2x|$  for  $0 \leq x \leq \frac{3}{2}\pi$ . [3 marks]*

- (b) Seterusnya, dengan menggunakan paksi yang sama, lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan  $|4 \sin 2x| - 1 = \frac{2x}{3\pi}$  untuk  $0 \leq x \leq \frac{3}{2}\pi$ .

Nyatakan bilangan penyelesaian itu. [3 markah]

*Hence, using the same axes, sketch a suitable straight line to find the number of*

*solutions for the equation  $|4 \sin 2x| - 1 = \frac{2x}{3\pi}$  for  $0 \leq x \leq \frac{3}{2}\pi$ .*

*State the number of solutions. [3 marks]*



- (a) Diberi  $\sin m \cos n = p$  dan  $\cos m \sin n = \frac{1}{p}$ , cari nilai  $\sin(m+n)\sin(m-n)$  dalam sebutan  $p$ . [2 markah]

*Given  $\sin m \cos n = p$  and  $\cos m \sin n = \frac{1}{p}$ , find the value of  $\sin(m+n)\sin(m-n)$  in terms of  $p$ . [2 marks]*

- (b) Lakar graf  $y = 1 + |\tan x|$  untuk  $0 \leq x \leq 2\pi$ . [4 markah]

*Sketch the graph of  $y = 1 + |\tan x|$  for  $0 \leq x \leq 2\pi$ . [4 marks]*

- (c) Seterusnya, dengan melakar satu graf yang sesuai pada paksi yang sama, tentukan bilangan penyelesaian yang memenuhi persamaan  $x(1 + |\tan x|) - 2\pi = 0$  untuk  $0 \leq x \leq 2\pi$ . [2 markah]

*Hence, by sketching a suitable graph on the same axes, determine the number of solutions that satisfy the equation  $x(1 + |\tan x|) - 2\pi = 0$  for  $0 \leq x \leq 2\pi$ . [2 marks]*

**FUNGSI TRIGONOMETRI**  
**MIMS 53**



(a) Buktikan bahawa  $\frac{\sin x}{1 - \cos x} = \operatorname{cosec} x + \cot x$ . [2 markah]

*Prove that*  $\frac{\sin x}{1 - \cos x} = \operatorname{cosec} x + \cot x$ . [2 marks]

(b) Lakar graf bagi  $y = -3 \cos 2x + 1$  untuk  $0 \leq x \leq \pi$ . Seterusnya, dengan menggunakan paksi yang sama, lakar satu garis lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan  $3 \cos 2x = -\frac{x}{\pi}$  untuk  $0 \leq x \leq \pi$ .

Nyatakan bilangan penyelesaian itu. [6 markah]

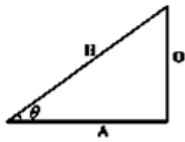
*Sketch the graph of  $y = -3 \cos 2x + 1$  for  $0 \leq x \leq \pi$ . Hence, by using the same axes, sketch a suitable straight line to find the number of solution for equation  $3 \cos 2x = -\frac{x}{\pi}$  for  $0 \leq x \leq \pi$ .*

*State the number of solutions.* [6 marks]

**FUNGSI TRIGONOMETRI**  
JAWAPAN



**YIK**



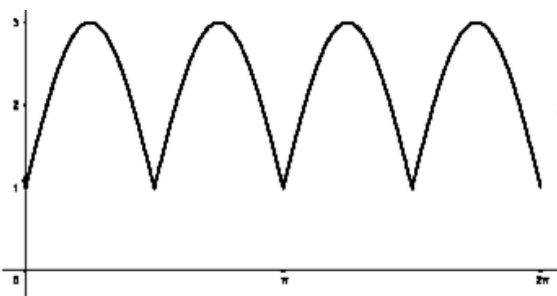
$$\sin \theta = \frac{O}{H}, \cos \theta = \frac{A}{H}$$

$$O^2 + A^2 = H^2$$

$$\frac{O^2}{H^2} + \frac{A^2}{H^2} = \frac{H^2}{H^2}$$

$$\left(\frac{O}{H}\right)^2 + \left(\frac{A}{H}\right)^2 = 1$$

$\sin^2 \theta + \cos^2 \theta = 1$ , terbukti



**SBP**

(a)  $\frac{2 \tan \theta}{1 - \tan^2 \theta}$

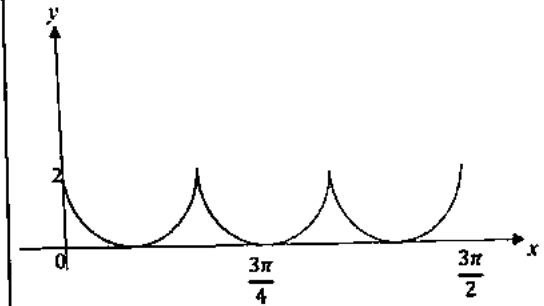
(i)  $\tan 2\theta$

(ii)  $\tan 2\theta = \frac{\sqrt{3}}{2}$

$\angle$  Rujukan =  $40.89^\circ$

$\theta = 20.45^\circ, 110.45^\circ, 200.45^\circ$

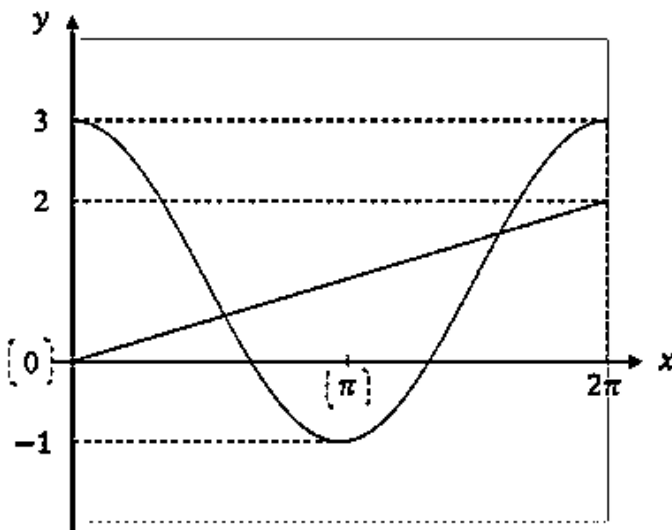
(b)



**MRSM**

$$\operatorname{cosec}^2 x = 1 + \cot^2 x \quad \text{or} \quad \sin^2 x + \cos^2 x = 1$$

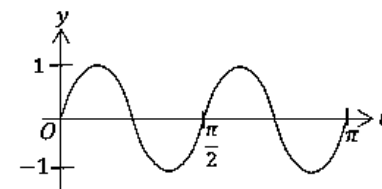
$$\cos^2 x$$



**SABK S1**

$$\sin 4\theta \cos \frac{\pi}{3} - \cos 4\theta \sin \frac{\pi}{3} + \sin 4\theta \cos \frac{\pi}{3} + \cos 4\theta \sin \frac{\pi}{3}$$

$$2 \sin 4\theta \left(\frac{1}{2}\right) = \sin 4\theta \quad (\text{terbukti})$$



Bentuk sin

2 kala

Amplitud

$$\pi (y) = mx - \pi$$

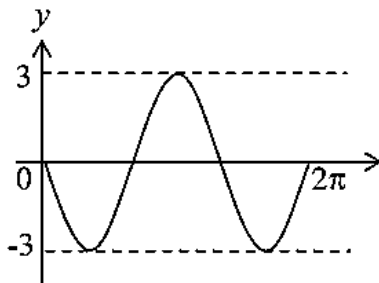
$$0 = \frac{m}{\pi} \left(\frac{\pi}{2}\right) - 1$$

$$m = 2$$

**FUNGSI TRIGONOMETRI**  
JAWAPAN

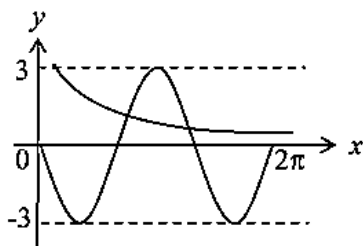


**SABK S3**



$$y = \frac{\pi}{x}$$

x	$\pi$	$2\pi$
y	1	$\frac{1}{2}$



2 bilangan penyelesaian

**SABK S2**

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2(25) = 1 - 2 (\sin 25)^2$$

$$= 1 - 2y^2$$

$$\sin (A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin (45-25) = \sin 45 \cos 25 - \cos 45 \sin 25$$

$$= (\sqrt{1-x^2})(\sqrt{1-y^2}) - xy$$

$$= (1-x^2)(1-y^2) - xy$$

$$\frac{2}{\cos x} = 2 \left( \frac{\sin x}{\cos x} \right) - \left( \frac{\cos x}{\sin x} \right)$$

$$\frac{2}{\cos x} = \frac{2\sin^2 x - \cos^2 x}{\cos x \sin x}$$

$$\frac{2\cos x \sin x}{\cos x} = 2\sin^2 x - \cos^2 x$$

$$2\sin x = 2\sin^2 x - (1 - \sin^2 x)$$

$$2\sin x = 3\sin^2 x - 1$$

$$3\sin^2 x - 2\sin x - 1 = 0$$

$$(3\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{3} \quad \sin x = 1$$

$$\alpha = 19.47 \quad \alpha = 90$$

$$x = 90^\circ, 199.47^\circ, 340.53^\circ$$

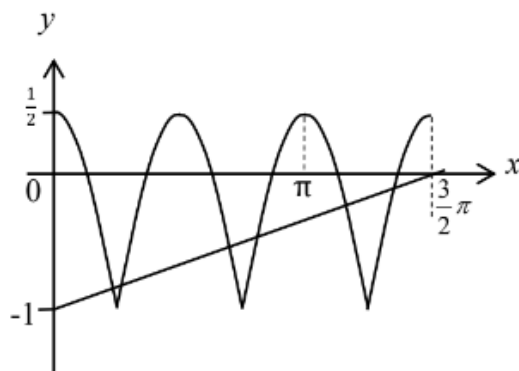
**SABK TRIAL**

Graf kosinus/ *Cosinus graph*

$1\frac{1}{2}$  kitaran / cycle

Amplitud =  $1\frac{1}{2}$  (Maksimum ;  $\frac{1}{2}$ , Minimum ; -1)

Mutlak **DAN** anjakan 1 unit ke bawah  
*Absolute AND move 1 unit down*

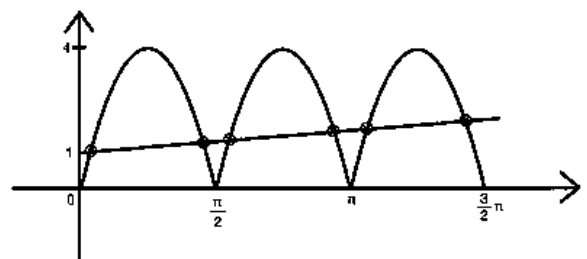


$$3(y+1) - 4x = -2x$$

Garis lurus / *Straight line*

6 bilangan penyelesaian / 6 no of solution

**MIMS S1**



Lakar garis lurus

$$y = \frac{2x}{3\pi} + 1$$

Bil. Penyelesaian = 6

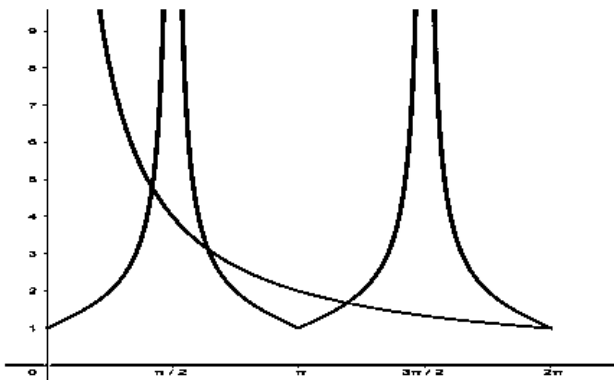
**FUNGSI TRIGONOMETRI**  
JAWAPAN



**MIMS S2**

$$\left(p + \frac{1}{p}\right)\left(p - \frac{1}{p}\right)$$

$$p^2 - \left(\frac{1}{p}\right)^2 \text{ OR } \frac{p^4 - 1}{p^2}$$



Shape of  $\tan x$

1 cycle for  $0 \leq x \leq 2\pi$

Modulus graph

shifted 1

Graf  $y = \frac{2\pi}{x}$

Bil penyelesaian = 4

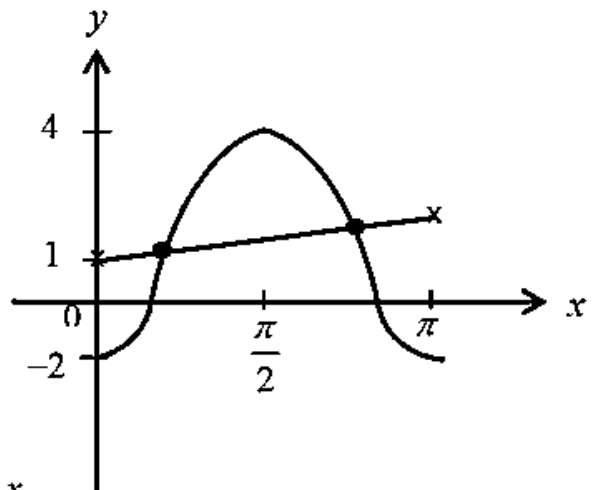
**MIMS S3**

$$\frac{\sin x(1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$$

$$\frac{\sin x + \sin x \cos x}{1 - \cos^2 x}$$

$$\frac{1 + \cos x}{\sin x} + \frac{\cos x}{\sin x}$$

kosek  $x$  + kot  $x$



$y = \frac{x}{\pi} + 1$

Bilangan penyelesaian = 2